

## **Fear vs. Greed: The Full Picture of the Autocorrelation in Market Volatility Index**

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## **Abstract**

After accounting for the size and sign of the changes in the VIX results, the OLS regressions show that the autocorrelation in the VIX is more complex than just being negative, it is actually positive following an extreme change in the VIX. More strikingly, the finding of a negative autocorrelation is driven by the negative changes in the VIX and autocorrelation following a positive change is statistically insignificant. The full picture of the autocorrelation is revealed in the results from the quantile regressions. They show that autocorrelation steadily falls in magnitude from lower to upper quantiles. Notably, following extreme changes in the VIX, autocorrelation changes from negative with a steadily falling magnitude in lower quantiles to zero and then becomes positive with a steadily increasing magnitude in higher quantiles. Finally, autocorrelation is clearly affected by the emergence of unexpected economic and financial crises and geopolitical events. However, the effect is short-lived.

## **Keywords**

Autocorrelation, Quantile Regression, Market Sentiment, Volatility Index

## **JEL classification**

C22, G14

## I. Introduction

Since its inception in 1993, the Chicago Board Options Exchange (CBOE) volatility index, or the VIX, has become the de facto volatility gauge that reflects market sentiment just as the S&P 500 is the de facto barometer of the health of the stock market. Beyond being a mere sentiment index, its importance is further solidified with the launch in 2004 and 2006, respectively, of the VIX futures and VIX options which have afforded investors indispensable tools for hedging and speculation. What is equally, if not more, significant is the role the VIX plays in the genuine effort by Wall Street to turn 'volatility' into an asset class.<sup>1</sup> Specifically, hedge funds, quant funds, and algorithmic trading systems have used these derivatives to generate returns by tracking the VIX and taking advantage of its structure. The introduction of exchange traded products (ETPs), particularly “short volatility ETPs” that bet on the VIX staying at low levels, has opened the door for many retail investors to participate in an investment strategy that previously has only been available to institutions and sophisticated investors. Given the substantial roles the VIX plays, how successful various market participants use it in making well-thought-out hedging, speculation, and asset allocation decisions depends on how well they understand the behavior of the VIX. To contribute to that better understanding, this paper examines the autocorrelation structure of the changes in the VIX over the period from 1990 to 2017.

A good understanding of the VIX is lacking and urgently needed among many retail investors who were lured to short volatility ETPs during the record 15-month market rally from October

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<sup>1</sup> See “Wall Street's effort to turn 'volatility' into an asset class is partly to blame for this market chaos” by Bob Pisani, 6 February 2018. (<https://www.cnn.com/2018/02/06/wall-streets-effort-to-turn-volatility-intan-asset-class-is-partly-to-blame-for-this-market-chaos.html>) and “Volatility - The Birth of A New Asset Class”. (<https://www.investopedia.com/articles/optioninvestor/05/vixindex.asp#ixzz57CtqgRtX>)

2016 to January 2018. While big institutional players and the sponsors of these ETPs are likely to have hedged their exposures, most retail investors, who own as much as four-fifth of the outstanding shares of some of these products, unfortunately were ignorant of the immense risk of these products.<sup>2</sup> The liquidation or halt of more than a dozen ETPs worldwide in the aftermath of devastating losses on Monday February 5, 2018 is undeniably a painful reminder for these investors of the dangers of ignorance.<sup>3</sup> To avoid repeating the same mistake, a better knowledge of the properties of the VIX is required and autocorrelation is a critical element due to its practical implications for investor returns.

Among the various issues surrounding the VIX that have been addressed in previous studies, its relationship with stock market returns is the most investigated.<sup>4</sup> The evidence of a negative relationship suggests a possibility of earning abnormal returns in the stock market if one can predict or better react to changes in market sentiment reflected in the changes in the VIX. The issue of predictability of the VIX is therefore of interest due to not only its implication for market efficiency but also its relevance for practitioners. Yet, as far as we know, for over 20 years nothing

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<sup>2</sup> See “Black Monday for VIX ETPs leaves retail players smarting” by Saqib Iqbal Ahmed, 6 February, 2018, Reuters Business News.

<sup>3</sup> Notable among them are XIV, the exchange traded note (ETN) of Credit Suisse, following a 95% loss in value, and Nomura’s Next Notes S&P500 VIX Short-Term Futures Inverse Daily Excess Return Index ETN after losing more than 80% in value.

<sup>4</sup> Specifically, Blair et al. (2001) examine the link between implied volatility and realized volatility; Fleming et al. (1995) examine the autocorrelation of the daily and weekly changes in the VIX; Whaley (2000), Brown and Cliff (2004), Giot (2005), Banergee et al. (2007), Cipollini and Manzini (2007), and Rubbaniy et al. (2014) investigate whether VIX can be used to predict stock market returns; Copeland and Copeland (1999) and Boscaljon et al. (2011) examine the effectiveness of the VIX in timing shifts for style asset allocation; Thrasher (2017) examines whether the dispersion of the VIX is a better predictor of future spikes in the VIX.

new has been advanced since the finding in Fleming et al. (1995) that daily changes in the VIX display a slight first-order autocorrelation and weekly changes exhibit significant mean reversion. To fill this void, we provide a comprehensive picture of autocorrelation by taking into account various unique characteristics of the VIX in the examination.

First, as widely observed by market participants and extensively examined by academics, e.g., Whaley (1993), the VIX experiences jumps that coincide with the market turmoil caused by unexpected economic and financial crises, and geopolitical events. This linkage to market turmoil explains why the VIX has been promoted and widely accepted as the most watched “fear gauge”—a term coined by Steven Sears, a Senior Editor and Columnist with Barron's and Barrons.com. The clear emphasis, and to some extent fixation, on the element of fear, rather than greed, is due to investors' inherent risk aversion. This aversion manifests itself in the elevated VIX level during times of uncertainty which is the result of investors willingness to pay more for protection. The heightened interest in the VIX in times of uncertainty justifies an examination that enhances our understanding of the behavior of the VIX at high levels.

Second, as interest in betting on the VIX remaining at low levels gains popularity evidenced by the record \$3.7 billion assets in short-VIX funds as of January 2018, the behavior of the VIX at low levels deserves the same, if not more, attention. Furthermore, from the perspective of behavioral finance, a good grasp on the behavior of the VIX requires one to pay equal attention to both high and low levels given that both extremes have been viewed as contrarian signals that can be used to identify market turning points. Many contrarians argue that extremely high (low) VIX levels result from indiscriminate selling (buying) by investors due to excessive pessimism (complacency). Hoping to profit from a contrarian strategy, they would go against the behavioral

biases underlying the extreme levels by buying (selling) stocks when the VIX is extremely high (low). Copeland and Copeland (1999) show that the VIX can be an effective timing tool for asset allocation which is consistent with this argument. Whaley (2000), Giot (2005), and Banerjee et al. (2007) similarly find the VIX to be a good predictor of stock index returns. The predictive power is particularly strong for extremely high or low levels of the VIX. To gain a better understanding of the VIX, we must examine the VIX at both high and low levels.

Third, analyzing both extremes of the VIX correlates to analyzing both positive and negative changes in the VIX. On the one hand, the apparent emphasis on the fear aspect of the VIX suggests that positive (not negative) changes resulting from increases (not decreases) in the VIX are more pertinent for the VIX as a sentiment barometer. On the other hand, from the viewpoints of speculation and asset allocation, negative changes in the VIX are just as important.

Finally, unlike stocks, bonds, and other tangible assets, the VIX, and volatility in general, has no intrinsic value. It cannot go to zero.<sup>5</sup> Repeatedly, following a sharp increase during market turmoil, it slowly drifts back down towards its mean, which, according to Bill Luby of VIX and More dating back to 1990, is 20.12. As reported later, the high kurtosis and positive skewness indicate clearly that changes in the VIX are not normally distributed. Consequently, to gain a better understanding of the VIX a detailed examination across the whole distribution, not merely the mean and standard deviation, is called for.

While ordinary least squares (OLS) regression has been the standard approach to estimating

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<sup>5</sup> The lowest level of 9.39% is recorded on December 11<sup>th</sup>, 2006 and highest level of 80.9 reached on November 20, 2008.

autocorrelation, it is incapable of reflecting the aforementioned rich nuances of the VIX due to the inherent limitations of OLS regression.<sup>6</sup> To overcome these shortcomings, we employ quantile regressions introduced by Koenker and Bassett (1978) to provide a full picture of the autocorrelation of changes in the VIX. Indeed, by accounting for the size and sign of the changes in the VIX, results from the OLS regressions show that autocorrelation in the VIX is richer than just being negative. It is actually positive following an extreme change in the VIX. More strikingly, it is negative only after a negative change; it is statistically insignificant after a positive change. The richness, as expected, is fully revealed in the results from quantile regressions. They show that autocorrelation steadily falls in magnitude from lower to upper quantiles. This new finding is interesting in view of the fixation of mass media and many investors on the increase, i.e., a positive change, in the VIX. Equally interesting, extreme changes in the VIX distinctly affect autocorrelation in such a way that autocorrelation changes from negative with a steadily falling magnitude in lower quantiles to zero, then to positive with a steadily increasing magnitude in higher quantiles. Finally, just like the attention of the market tends to be heightened as unexpected economic or geopolitical events emerge, we find that autocorrelation is affected by these

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<sup>6</sup> Specifically, OLS regression, also referred to as “mean regression,” gives us an estimate of the autocorrelation coefficient that only looks at the mean changes in the VIX. As such, it ignores the extreme changes in the VIX rendering it incapable of providing a full picture of the autocorrelation across the whole distribution of the changes in the VIX. Furthermore, given that there are no theories based on which one can say with confidence whether autocorrelation exists and what sign it takes, the issue of the autocorrelation in the changes in the VIX is entirely an empirical one. It can be zero, positive, or negative and it may vary across the whole distribution of changes in the VIX. If it varies evenly from negative to positive along the distribution, then positive autocorrelation is cancelled out by negative autocorrelation, leading to a zero autocorrelation on average. This is another potential failure of OLS regression in depicting the full picture of the autocorrelation. Finally, the well-known influence of outliers in OLS regressions means that OLS estimates tend to be pulled towards the outliers potentially resulting in a biased estimate, which can be either positive or negative depending on whether the outliers are located in the positive or negative end of the distribution.

unexpected events. The effect, however, is short-lived.

The remainder of the paper is organized as follows: Section II discusses the data and methodology, Section III presents the results, and Section IV concludes the paper.

## **I. Data and Methodology**

The data used in this study consists of the daily closing values of the VIX obtained from the VIX Options and Futures Historical Data of the Federal Reserve Economic Data. The sample covers the period from January 2, 1990 to March 15, 2017. To examine the autocorrelations for various quantiles, we start with the first-order quantile autocorrelation model, called the Basic Model, shown below:

$$Q(\tau|F_{t-1}) = \alpha(\tau) + \beta(\tau)\Delta VIX_{t-1} \quad (1)$$

where  $\Delta VIX_t$  is the percentage change in the VIX on day  $t$  calculated as  $(VIX_t - VIX_{t-1})/VIX_{t-1}$ ,  $Q(\tau|F_{t-1})$  is the conditional  $\tau$ -th quantile of  $\Delta VIX_t$ , and  $F_{t-1}$  denotes the information set of the VIX's past changes.  $\beta(\tau)$ , the coefficient for  $\Delta VIX_{t-1}$ , is the estimate for autocorrelation. For comparison, we also estimate the first-order autocorrelation based on ordinary least square (OLS) regressions.

To examine the differential effect of extreme past changes in the VIX on autocorrelation, we extend the Basic Model to the Size Model shown below by adding two terms that involve an indicator variable,  $I(|\Delta VIX_{t-1}| > \Delta VIX^q)$ :

$$\begin{aligned}
Q(\tau|F_{t-1}) &= [\alpha_i(\tau) + \lambda_i(\tau)I(|r_{t-1,i}| > r^q)] + [\beta_i(\tau)r_{t-1,i} + \gamma_i(\tau) r_{t-1,i}I(|r_{t-1,i}| > r^q)] \\
&= [\alpha_i(\tau) + \lambda_i(\tau)I(|r_{t-1,i}| > r^q)] + [\beta_i(\tau) + \gamma_i(\tau)I(|r_{t-1,i}| > r^q)] r_{t-1,i} \quad (2)
\end{aligned}$$

where  $I(|\Delta VIX_{t-1}| > \Delta VIX^q)$  equals one if the absolute value of  $\Delta VIX_{t-1}$ ,  $|\Delta VIX_{t-1}|$ , is greater than a threshold  $\Delta VIX^q$  and zero otherwise. Following Baur et al. (2012),  $\Delta VIX^q$  is defined as the 95<sup>th</sup> percentile of past  $\Delta VIX$ s. From equation (2), the effect of an extreme past change on current change has two components: (1) a shift in the intercept, captured by  $\lambda(\tau)$ , and (2) a change in the slope, measured by  $\gamma(\tau)$ . By including the intercept component, we account for any possible upward or downward shifts in the intercept due to an extreme past change in the VIX. In doing so, we avoid the potential drawback of a biased estimate of the slope because of the failure to account for the effect on the intercept of past extreme changes in the VIX. Since the estimate for the autocorrelation following an extreme change is  $\beta(\tau) + \gamma(\tau)$ , when  $\gamma(\tau)$  is statistically significant, the magnitude of  $\beta(\tau) + \gamma(\tau)$  is greater than  $\beta(\tau)$  if  $\beta(\tau)$  and  $\gamma(\tau)$  are of the same sign. If they are of opposite signs, then the magnitude is smaller as the positive value cancels out the negative value resulting in a net value that is smaller in magnitude than that of  $\beta(\tau)$ .

To measure the effect of the sign of past changes on current changes in the VIX, we add an indicator variable  $I(\Delta VIX_{t-1} < 0)$  to the Basic Model to arrive at the following Sign Model:

$$Q(\tau|F_{t-1}) = [\alpha_i(\tau) + \rho_i(\tau)I(\Delta VIX_{t-1} < 0)] + [\beta_i(\tau)r_{t-1,i} + \delta_i(\tau) r_{t-1,i}I(\Delta VIX_{t-1} < 0)] \quad (3)$$

where  $I(\Delta VIX_{t-1} < 0)$  takes on a value of one if  $\Delta VIX_{t-1}$  is negative. Similar to equation (2), both

the intercept and the slope are allowed to be affected by past negative changes in the VIX. The net effect of the negative changes in the VIX on autocorrelation similarly depends on whether the signs of  $\beta(\tau)$  and  $\delta(\tau)$  are the same. If they are, then the magnitude of the autocorrelation increases, otherwise, it decreases. It is worth noting that while Baur et al. (2012) and Ceretta et al. (2012) use regression models similar to those in equations 1 – 3, their models do not include the intercept component in equations 2 and 3. By including the intercept component, we account for any possible upward or downward shifts in the intercept due to an extreme or a negative change in the VIX.

Finally, an examination of extreme changes in the VIX would be incomplete without an analysis of the spikes that grab the attention of the market during times of turmoil surrounding major economic or geopolitical events. To perform this examination, we extend the Basic Model by incorporating an indicator variable. Denoted as  $I(t)$ , it takes on a value of one if  $t$  is within the four weeks, as an approximation of one month, or within 12 weeks, an approximation of three months, after the occurrence of an event and zero otherwise. The selection of these two time periods is meant to capture any short- or mediate-term effects, if they exist. Given that the VIX is purported to measure the expected market volatility over the next 30 days, one should not expect to see any effect of these events for periods longer than three months. The selection of four weeks, instead of a shorter period, is due to the consideration that sufficient observations are needed to accurately detect the effect. As shown below in Equation (4), this extended model is called the Event Model:

$$Q(\tau|F_{t-1}) = [\alpha_i(\tau) + \lambda_i(\tau)I(t)] + [\beta_i(\tau)r_{t-1,i} + \theta_i(\tau) r_{t-1,i}I(t)] \quad (4)$$

Like equations (2) and (3), both the intercept and the slope are allowed to change and the magnitude of the autocorrelation in the pre-crisis period is greater (smaller) if  $\beta_i(\tau)$  and  $\theta_i(\tau)$  are of the same (opposite) sign. Without a preconceived notion of what constitutes a major event, we select four events, which clearly are not meant to be exhaustive, for this exploration. These events that have caught the attention of the market when they occurred are: the 1997 Asian financial crisis, 2007 subprime financial crisis, the first Persian Gulf war, and the declaration of a Caliphate on June 29, 2014 by the Islamic State in Iraq and Greater Syria (ISIS). The first two are undoubtedly financial events whose impacts rippled throughout the world. On the other hand, the last two are geopolitical events that nevertheless impacted the economy indirectly via oil prices and consumer confidence. Except for the declaration of a Caliphate by ISIS that caught the world off guard on June 29, 2014, the two economic events and the first Persian Gulf war emerged with many warning signs—in the case of the first Persian Gulf war it was the invasion of Kuwait by Saddam Hussein on August 2, 1990—until the impacts were fully felt. It is therefore prudent that we use the month when these three events became headline news for the 4-week period, and as the first month of the 12-week period. Based on this criteria, the month of the first three events are respectively September of 2008 for the subprime crisis, October of 2009 for the Eurozone debt crisis, and August of 1990 for the first Persian Gulf war. For ISIS, the 4-month and 12-month periods start on June 29, 2014.

### **III. Results**

Table 1 reports the summary statistics of the daily changes in the VIX. During the sample period, there are a total of 6,851 daily changes in the VIX. With a skewness value of 1.2764 and

kurtosis value of 10.1434, the daily changes in the VIX are not normally distributed, which is confirmed by the Jarque-Bera statistic. The violation of normal distribution in changes in the VIX means that ordinary least squares regressions may produce biased estimates due to outliers, justifying the use of quantile regressions to adequately address this methodological issue. This is true for each of the deciles. With a median value of -0.0033, it is not surprising to see that the first five deciles are all negative while the last five are all positive except for decile 6 which reports a minimum of -0.0032. Interestingly, the standard deviations across the deciles exhibit a V-shape pattern. At decile 1 with the value 0.032, the standard deviation steadily decreases to 0.0035 at decile 5. The trend reverses at decile 6, which has a value of 0.0036, and the standard deviation steadily increases to the highest value of 0.0695 at decile 9.

Insert Table 1 about here

In presenting the results, we report the estimates and their p-values. To see the trend in the magnitude across quantiles, we also present the estimates in figures. The results for the Basic Model reported in Table 2 show that the OLS estimate for the intercept,  $\alpha$ , is 0.0021, which is statistically significant at the one percent level.  $\beta$ —the estimate for autocorrelation—has a value of -0.0798, which is significant at the one percent level. This negative autocorrelation is consistent with the finding in Fleming et al. (1995) of a negative first-order autocorrelation on daily changes in the VIX. This tendency of the changes in the VIX to change sign is in line with the mean-reversion characteristics of the VIX. Behind the positive OLS-estimated  $\alpha$ , we see clear variations in  $\alpha$  among the quantiles. It steadily increases in value from a negative -0.0674 at the 10% quantile to -0.0036 at the 50% quantile. It then becomes positive, with a value of 0.0746, at the 60% quantile and continues to increase to 0.0092 at the 90% quantile. All of these estimates are significant at

the one percent level. Contrasting with these variations,  $\beta$  is negative and remains so across all quantiles. Its magnitude steadily decreases from -0.106 at the 10% quantile to -0.0251 at the 90% quantile. It is statistically significant at the one percent level for quantiles 10% to 70%. For the 80% quantile, it is significant with a p-value of 0.058. For the 90% quantile it is not significant.

The variations across quantiles clearly indicate that underneath the OLS estimate of a negative autocorrelation in the changes in the VIX, the autocorrelation steadily decreases in magnitude from the lower to upper quantiles. Therefore, while there is a tendency for a reversal in sign following a prior change in the VIX, the tendency is stronger in the lower quantiles. Given that lower quantiles are changes that are negative, the greater magnitude in autocorrelation in lower quantiles suggests that when the VIX is falling fast mean-reversion is more pronounced. On the other hand, since the upper quantiles are positive changes in the VIX, corresponding to when the VIX experiences large increases, the continuously falling magnitude in autocorrelation suggests that the VIX is harder to predict amidst market turmoil.

Graphically, Figure 1 clearly shows autocorrelation is negative and that there is a generally upward trend from being large negative in lower quantiles to small negative in upper quantiles.

Insert Table 2 about here

Insert Figure 1 about here

Table 3 reports the results for the Size Model of equation (3) that examines the effect of a past extreme change in the VIX on autocorrelation. From the OLS regression,  $\alpha$  has a value of 0.002 and  $\beta$  has a value of -0.0928 and both are statistically significant at the one percent level. While

the estimate for  $\alpha$  is similar to the estimate of 0.0021 in the Basic Model, the  $\beta$  estimate of -0.0928 is larger in magnitude than that from the Basic Model of -0.0798. The similarity in  $\alpha$  estimates is consistent with the statistically insignificant  $\lambda$  estimate that measures the differential effect on the intercept of an extreme change in the VIX. The statistical insignificance of  $\lambda$  suggests no shift in the intercept following an extreme change in the VIX. This is not the case for autocorrelation, however. With  $\gamma$  having a statistically significant value of 0.1758, autocorrelation is shown to be different after an extreme change in the VIX. In fact, it changes sign from negative to positive since the autocorrelation coefficient value after an extreme change is measured by the sum of  $\gamma$  and  $\beta$ , which is 0.083. Therefore, following an extreme change in the VIX, there is a tendency for the sign of the next change in the VIX to stay the same, different from the mean-reversion result from the Basic Model.

Behind this new OLS finding, more interesting details are revealed in the quantiles results. First,  $\beta$  is negative and statistically significant across all quantiles at the one percent level except for the 90% quantile which is at the ten percent level. Second, with a minor exception at the 80% quantile, it steadily decreases in magnitude from the lower to the upper quantiles. Third, in contrast to this steady pattern that is similar to the pattern in the results from the Basic Model, there are notable variations in  $\gamma$  across quantiles. Except for the 10% and 20% quantiles, it is positive for the remaining quantiles. For quantiles 10% to 50%, it is statistically insignificant. Beyond that, its p-value is 0.03, 0.01, or smaller. These statistically significant and positive coefficients among the upper quantiles explain the OLS estimate of a positive  $\gamma$  of 0.178. The conclusion based on these variations in  $\gamma$  is that autocorrelation following an extreme change in the VIX varies with the quantiles. To help see that, we report in Panel B the sum of  $\beta$  and  $\gamma$ . When  $\beta$  and  $\gamma$  are of the same sign, autocorrelation is more pronounced following extreme returns whereas autocorrelation is

smaller in magnitude if  $\beta$  and  $\gamma$  are opposite in sign. In adding  $\gamma$  to  $\beta$ , we treat any  $\beta$  or  $\gamma$  that is insignificant as zero if the other one is significant and use “-” to indicate the case when both  $\beta$  and  $\gamma$  are insignificant. By treating insignificant  $\gamma$  estimates from quantile 10% to 50% as zero,  $\beta+\gamma$  in fact equals to  $\beta$  in these lower quantiles, which is negative with a magnitude that is steadily falling. However, for quantiles 60% to 90%,  $\beta+\gamma$  is positive and steadily increases in value, from 0.0112 at the 60% quantile to 0.3736 at the 90% quantile.

These results indicate that while autocorrelation following an extreme change in the VIX is negative for lower quantiles, it becomes positive and increases in value among upper quantiles where positive changes in the VIX are. This novice finding of a positive and increasing autocorrelation among upper quantiles indicates that following an extreme change in the VIX, a positive change will be followed by a positive change, suggesting a continuing trend for the VIX to remain elevated amidst market turmoil.

Insert Table 3 about here

Insert Figure 2 about here

Results for the Sign Model that measures the differential impacts between positive and negative past changes in the VIX are reported in Table 4. Under the OLS,  $\alpha$  has a value of 0.0505 which is statistically significant and  $\beta$  has a statistically insignificant value of 0.0186. This result of an insignificant  $\beta$  contrasts sharply with the finding of a significantly negative  $\beta$  from the Basic Model. The reason for this new finding apparently lies in  $\delta$ , the coefficient measuring the differential impact of a negative change in the VIX on autocorrelation. Its value of -0.1022 is significant at a level better than one percent, indicating a negative autocorrelation following a

negative change. These two results together mean that a significant autocorrelation is present only after a negative change in the VIX and its sign is negative while autocorrelation following a positive change in the VIX is insignificant. This conclusion is confirmed in quantile results. Across the quantiles, the  $\beta$  estimates exhibit no clear pattern and, more importantly, are statistically insignificant. On the other hand,  $\delta$  is negative and generally decreases in magnitude from lower to upper quantiles. It is significant at better than one percent level for the 10% to 60% quantiles, significant with a p-value of 0.09 and 0.03, respectively, for the 70% and 80% quantiles, but insignificant at the 90% quantile. This negative and steadily falling pattern of  $\delta$  mirrors the pattern of  $\beta$  from the Basic Model, suggesting the results from the Basic Model are in fact the results for the negative changes in the VIX.

The above result that autocorrelation is only present in the negative changes is refreshing considering the fixation by the mass media on the jumps, which are clearly huge positive changes, in the VIX. The insignificant autocorrelation of positive changes suggests the lack of a predictable pattern following a positive change. Given that this happens when the VIX increases, it means no one can predict what will follow an increase in the VIX. On the other hand, the significant and negative autocorrelation following negative changes in the VIX indicates a tendency for reversal. Furthermore, the magnitude of the autocorrelation steadily decreases from the largest value at the lowest quantile to the smallest value at the uppermost quantile indicating that the more negative a change is in the VIX, the more likely the next change will be of a different sign, which is positive. Since a negative change occurs when the VIX decreases in value, i.e. when fear in the market subsides, the negative autocorrelation of negative changes means a tendency for the VIX to zigzag in the mean-revision process.

Insert Table 4 about here

Insert Figure 3 about here

As an integral part of the examination of extreme changes in the VIX, the impact of major events on autocorrelation is examined next using the Event Model. The results that include all four events are reported first in Table 5. In Panel A, the results for one month after the events are reported. Under OLS,  $\beta$  has a value 0.019 which is statistically significant at better than one percent level. The positive sign is interesting since it contrasts the negative result from the Basic model. The estimate for the impact of the event on autocorrelation in the immediate post-event period,  $\theta$ , is not statistically significant which is similar to the OLS result in Table 4 for the Sign Model. On the other hand, the results for the quantiles are similar to those for the Basic Model. Across the quantiles,  $\beta$  is negative with a steadily falling magnitude from -0.12 at quantile 0.1 to -0.024 at quantile 0.9. Except for quantile 0.9, it is statistically significant, at the one percent level for quantiles 0.1 to 0.7 and at the 10 percent level for quantile 0.8. For  $\theta$ , it is negative for quantiles 0.1 to 0.8. However, it is significant only for quantiles 0.1 and 0.8, with a respective value of -0.131 and -0.184, which are significant at the one and five percent levels, respectively. For quantile 0.9,  $\theta$  is positive, with a value of 0.317 and is statistically significant at the one percent level. This positive autocorrelation at quantile 0.9 explains the positive autocorrelation at quantile 0.9 reported in Table 3. Adding together the significant  $\theta$  and  $\beta$ ,  $\beta + \theta$ , the autocorrelation for the one-month event period is a significant -0.251 at quantile 0.1, -0.246 at quantile 0.6, and 0.317 at quantile 0.9. It is interesting to note that while  $\beta + \theta$  steadily falls in magnitude from quantile 0.1 to 0.5, it increases markedly at quantile 0.6, but then resumes the falling trend and turns positive at quantile 0.9. Continuing on to the results in Panel B for the three-month period after the event, one sees that the estimates for  $\beta$  based on OLS and across quantiles are effectively the same as those in

Panel A for the one-month period after the event. For  $\theta$ , only quantile 0.1 reports a significant value of -0.146 and p-value of 0.015, suggesting that after three months, the events no longer have a significant effect for all quantiles except for quantile 0.1.

Insert Table 5 about here

Insert Figure 4 about here

While lumping all four events together allows us to explore any changes in the autocorrelation, the examination of each event individually may yield further insights. In particular, we will be able to see if the results for economic events, which should have clear and direct impacts on the economy, differ from those for geopolitical events, whose effects on the economy are indirect. We first look at the two economic events and report the results in Table 6. Results for the 2007 subprime financial crisis are reported in Part 1. For the one-month period after the event, Panel A shows that the OLS estimates for  $\beta$  and  $\theta$  are 0.019, significant at the one percent level, and -0.218, significant at the ten percent level respectively. The sum of the two,  $\beta + \theta$ , is -0.199. The contrast of a significant  $\theta$  here with the insignificant  $\theta$  in Table 5 suggests the subprime crisis has a unique impact compared with other three events. Across the quantiles, the estimates for  $\beta$  are negative with a steadily falling magnitude, similar to those for all events. For  $\theta$ , the estimates do not show any trend in magnitude given that the magnitude of -0.166 for quantile 0.3 is greater than that of -0.088 for quantile 0.2. Similarly, quantiles 0.5 to 0.9 all have a magnitude greater than quantiles 0.1 to 0.4. Adding  $\theta$  to  $\beta$ ,  $\beta + \theta$ , we see that across all quantiles, the autocorrelation during the one-month period of the crisis is negative. While there is no steady trend, the upper quantiles, quantiles 0.6 to 0.9, have an autocorrelation of -0.374 for quantile 0.7 and -0.642 for quantile 0.9. These large negative and significant autocorrelations indicate a greater tendency for the VIX to zigzag

when it experiences large positive changes amidst the turmoil of the crisis.

For the three-month period following the event, the OLS result is similar to the result for all events reported in Table 5. This is also true for quantiles 0.1 to 0.5 whose  $\theta$  is not significant. For quantiles 0.6 to 0.8, however,  $\theta$  is negative and significant at the one percent level for quantiles 0.6 and 0.7 and close to the one percent level for quantile 0.8. Adding  $\theta$  to  $\beta$ , we see that  $\beta + \theta$  autocorrelation in the three-month period following the event is negative and falls in magnitude from quantiles 0.1 to 0.5. It then increases in magnitude from -0.204 at quantile 0.6 to 0.8. The autocorrelation for quantile 0.9, however, is not significant. Together, these results suggest that while there is a tendency for a reversal when the VIX is relatively high, there is no predictable direction for the VIX at the highest quantile.

From the results for the Eurozone debt crisis reported in Part 2 of Table 6 we see that during the one-month period after the crisis, the OLS estimates are similar to those for all events. Across quantiles,  $\theta$  is significant, at the ten percent level, only for quantiles 0.5 and 0.6 with respective values of 0.249 and 0.261. Adding these two to  $\beta$ , we have an autocorrelation of 0.175 for quantile 0.5 and 0.193 for quantile 0.6 while the rest of the quantiles report no difference in autocorrelation. For the three-month period after the crisis, the results in Panel B show that quantile 0.3 has a positive  $\theta$  of 0.12 which is significant at a level slightly higher than one percent and quantile 0.9 has a negative  $\theta$  of -0.279 which is significant at the one percent level. The resulting  $\beta + \theta$  shows that autocorrelation of the VIX in the three-month period after the Eurozone debt crisis is the same as the rest of the time except for quantile 0.3 which is positive and quantile 0.9 which has the highest, in magnitude, negative autocorrelation.

Insert Table 6 about here

Insert Figure 5 about here

Comparing the two events, we see that the subprime crisis has led to a change in autocorrelation in more quantiles than the Eurozone debt crisis has: five versus two in the one-month period after the event and three versus two in the three-month period after the event. This difference appears to reflect the devastating impact of the subprime crisis on global economy while the fallout of the Eurozone debt crisis was relatively contained. Following these two economic events, the results for geopolitical events are reported in Table 7. Panel A of Part 1 shows that during the one-month period after the First Persian Gulf war, the OLS result remains the same as that for all events reported in Table 5. Results for the quantiles show that  $\theta$  is significant for quantiles 0.1, 0.2, 0.6, and 0.9, with respective values of 0.211 and a significance level of one percent, 0.076 and significance level of five percent, -0.108 and significance level of one percent, and 0.217 and significance level of one percent. The resulting  $\beta+\theta$ 's show that during the one-month period after the event autocorrelation is positive at quantiles 0.1 and 0.9 and is negative with a steadily falling magnitude from quantiles 0.2 to 0.5. They also show that it increases in magnitude at quantile 0.6, then continues to fall in quantiles 0.7 and 0.8. For the three-month period after the event, only quantile 0.6 reports a significant  $\theta$ , which has a value of -0.101 and is significant at the one percent level. The resulting  $\beta+\theta$ 's therefore suggests that except for quantile 0.6 which reports an increase in magnitude, autocorrelation during the three-month period after the event does not differ from other times.

Results for the declaration of a Caliphate by ISIS are reported in Part 2. In Panel A we see that under OLS  $\theta$  has a value -0.346 which is significant at the one percent level. The resulting  $\beta$

$\theta$  has a value of -0.227, indicating a negative autocorrelation during the one-month period after the effect. Across the quantiles,  $\theta$  is statistically significant, all at the one percent level, for quantiles 0.1, 0.4, 0.5, and 0.8. The resulting  $\beta + \theta$ 's show that autocorrelation in the one-month period following the event is negative for all quantiles except quantile 0.9. The magnitude falls from quantiles 0.1 to 0.3, increases at quantiles 0.4 and 0.5, decreases in quantiles 0.6 and 0.7, and then rises again at quantile 0.8. For the three-month period, the only significant  $\theta$  appears at quantile 0.1 with a value of -0.184 and is significant at the one percent level. This raises the autocorrelation considerably at quantile 0.1 during the three-month period after the event, while the rest of the quantiles remain unchanged.

Insert Table 7 about here

Insert Figure 6 about here

Comparing the results for the two geopolitical events, we see that during the one-month period following both events, four quantiles report a significant change in autocorrelation. Similarly, during the three-month period, only one quantile reports a significant change in autocorrelation. Overall, the results from the Event Model show some changes in autocorrelation after the event. In view of the small number of observations of the event periods examined relative to the number of observations in the whole sample, four and twelve weeks relative to 27 years, the existence of significant changes suggests genuine impacts of these events on the VIX. Given that there are more quantiles reporting significant changes in the one-month period than in the three-month period, the impacts of the events appear to be stronger immediately after the event and subside as time goes by. Finally, more significant changes are reported in the much lower and upper quantiles, suggesting the impacts of the events are stronger for the more extreme negative

and positive changes in the VIX.

#### **IV. Conclusion**

The recent fiascos experienced by many exchange traded products that bet on the VIX staying at low levels is a painful reminder of the disastrous consequences of being unaware of or willfully ignoring the behavior of the VIX, providing a timely and well-justified motivation for the examination in this study of autocorrelation in the VIX. The examination yields results that extend the current understanding of autocorrelation in the VIX well beyond the finding of a negative autocorrelation in the changes in the VIX provided over 20 years ago by Fleming et al. (1995). Notably, after accounting for the size and sign of the change in the VIX, the OLS results show that autocorrelation following an extreme change in the VIX is positive and autocorrelation following a positive change in the VIX is insignificant. Upon further examination based on quantile regressions, we unveil in more detail the variations behind these size- and sign-based results.

First, we show that behind the OLS finding of a positive autocorrelation following an extreme change is a V-shaped autocorrelation pattern. Autocorrelation is negative in lower quantiles with the magnitude steadily falling from the lowest quantile to middle quantiles. It then becomes positive and steadily increases in value with the uppermost quantile exhibiting the highest autocorrelation. This finding indicates that in an environment of negative changes in the VIX, i.e., when the VIX is subsiding, an extreme change in the VIX tends to be followed by a reversal and the tendency is stronger the more negative the environment is. On the other hand, amidst positive changes in the VIX, i.e., when the VIX is spiking, an extreme change in the VIX tends to be

followed by a continuation in the same direction and the more positive the change is, the stronger the tendency is.

Second, along with the result that autocorrelation following a positive change in the VIX is statistically insignificant is the finding of a negative autocorrelation following a negative change. Refreshingly, this means that the finding of a negative autocorrelation is driven by negative changes in the VIX and positive changes have nothing to do with it. Furthermore, the quantile results show that the magnitude of autocorrelation steadily falls from extremely negative changes in the lower quantiles to extremely positive changes in the upper quantiles. We also find autocorrelation is affected following financial crisis and geopolitical events. In view of the aforementioned variations in autocorrelation based on the size and sign of the changes in the VIX, this finding is to be expected. It is also not surprising to find that the effect is greater in the immediate one-month period than the three-month period following the event.

These fresh findings have clear implications for short-term speculation considering the fixation in the mass media on the spikes, i.e. positive changes, in the VIX. The finding of insignificant autocorrelation following a positive change suggests that as the VIX rises in uncertain times, the direction of the VIX becomes less predictable. On the other hand, the evidence that it is the negative changes that are responsible for the finding of a negative autocorrelation suggest that as the VIX falls and reverts back to its mean, it tends to zigzag.

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**Table 1 Summary Statistics of Changes in the VIX, 1990 - 2017**

	Whole	Decile									
	Sample	1	2	3	4	5	6	7	8	9	10
Observations	6851	685	685	685	685	685	685	685	685	685	686
Mean	0.0020	-0.0990	-0.0548	-0.0358	-0.0215	-0.0092	0.0030	0.0162	0.0327	0.0574	0.1310
Median	-0.0033	-0.0901	-0.0541	-0.0356	-0.0212	-0.0092	0.0030	0.0159	0.0325	0.0569	0.1090
Maximum	0.6422	-0.0681	-0.0440	-0.0283	-0.0152	-0.0033	0.0092	0.0238	0.0432	0.0747	0.6422
Minimum	-0.2957	-0.2957	-0.0681	-0.0440	-0.0283	-0.0151	-0.0032	0.0092	0.0239	0.0432	0.0748
Std. Dev.	0.0652	0.0320	0.0069	0.0046	0.0037	0.0035	0.0036	0.0043	0.0057	0.0089	0.0695
Skewness	1.2764	-2.1598	-0.2473	-0.0932	-0.1332	-0.0374	-0.0245	0.0869	0.1691	0.2272	2.9191
Kurtosis	10.1434	9.4699	1.8913	1.8131	1.8341	1.7941	1.8069	1.7587	1.7427	1.8734	14.4874
Jarque-Bera	16426.5	1727.3	42.1	41.2	40.8	40.7	40.7	44.8	48.4	42.1	4746.1
P Value	0.0000 <sup>a</sup>										

Note: Significance level is indicated by a for 1% level.

**Table 2 Coefficient Estimates of the Basic Model, Eq(1), of Daily Changes in the VIX**

Quantile	$\alpha$		$\beta$	
	Coeff.	P-Value	Coeff.	P-Value
OLS	0.0021 <sup>a</sup>	0.0010	-0.0798 <sup>a</sup>	0.0000
0.1	-0.0674 <sup>a</sup>	0.0000	-0.1206 <sup>a</sup>	0.0000
0.2	-0.0440 <sup>a</sup>	0.0000	-0.1090 <sup>a</sup>	0.0000
0.3	-0.0283 <sup>a</sup>	0.0000	-0.1043 <sup>a</sup>	0.0000
0.4	-0.0155 <sup>a</sup>	0.0000	-0.0854 <sup>a</sup>	0.0000
0.5	-0.0036 <sup>a</sup>	0.0000	-0.0708 <sup>a</sup>	0.0000
0.6	0.0092 <sup>a</sup>	0.0000	-0.0625 <sup>a</sup>	0.0010
0.7	0.0235 <sup>a</sup>	0.0000	-0.0555 <sup>a</sup>	0.0010
0.8	0.0434 <sup>a</sup>	0.0000	-0.0469 <sup>c</sup>	0.0580
0.9	0.0746 <sup>a</sup>	0.0000	-0.0251	0.2970

Note: Significance levels are indicated by the superscript on the coefficient estimate: a for 1% level, b for 5% level, and c for 10% level.

**Table 3 Coefficient Estimates of Size Model, Eq(2), of Daily Changes in the VIX**

**Panel A: Coefficient Estimates**

Quantile	$\alpha$		$\beta$		$\lambda$		$\gamma$	
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value
OLS	0.0020 <sup>a</sup>	0.0100	-0.0928 <sup>a</sup>	0.0000	0.0026	0.4700	0.1758 <sup>a</sup>	0.0000
10%	-0.0667 <sup>a</sup>	0.0000	-0.1157 <sup>a</sup>	0.0000	-0.0139 <sup>a</sup>	0.0100	-0.1138	0.2000
20%	-0.0435 <sup>a</sup>	0.0000	-0.1110 <sup>a</sup>	0.0000	-0.0070	0.0800	-0.0356	0.6900
30%	-0.0281 <sup>a</sup>	0.0000	-0.1074 <sup>a</sup>	0.0000	-0.0032	0.3900	0.0096	0.9200
40%	-0.0150 <sup>a</sup>	0.0000	-0.0935 <sup>a</sup>	0.0000	-0.0061 <sup>b</sup>	0.0500	0.0496	0.3300
50%	-0.0031 <sup>a</sup>	0.0000	-0.0807 <sup>a</sup>	0.0000	-0.0056 <sup>c</sup>	0.0800	0.0539	0.2000
60%	0.0097 <sup>a</sup>	0.0000	-0.0725 <sup>a</sup>	0.0000	-0.0047	0.2700	0.0837 <sup>b</sup>	0.0300
70%	0.0236 <sup>a</sup>	0.0000	-0.0653 <sup>a</sup>	0.0000	-0.0001	0.9900	0.1499 <sup>a</sup>	0.0100
80%	0.0433 <sup>a</sup>	0.0000	-0.0675 <sup>a</sup>	0.0100	0.0088	0.3600	0.2815 <sup>a</sup>	0.0000
90%	0.0737 <sup>a</sup>	0.0000	-0.0431 <sup>c</sup>	0.0900	0.0236	0.3700	0.4267 <sup>b</sup>	0.0300

**Panel B: Autocorrelation Without ( $\beta$ ) and Following ( $\beta+\gamma$ ) An Extreme Change in the VIX<sup>1</sup>**

Quantile	Following An Extreme Change	
	No	Yes
	$\beta$	$\beta+\gamma$
OLS	-0.0928	0.0830
10%	-0.1157	-0.1157
20%	-0.1110	-0.1110
30%	-0.1074	-0.1074
40%	-0.0935	-0.0935
50%	-0.0807	-0.0807
60%	-0.0725	0.0112
70%	-0.0653	0.0846
80%	-0.0675	0.2140
90%	0.0737	0.3836

Notes:

1. In Panel B only  $\beta$  and  $\theta$  that are significant, at a level of at least 10%, are used in the reported  $\beta+\gamma$ ; insignificant  $\beta$  or  $\gamma$  are treated as zero and “-” is used to indicate the case when both  $\beta$  and  $\gamma$  are insignificant.
2. Significance levels are indicated by a for 1% level, b for 5% level, and c for 10% level.

**Table 4 Coefficient Estimates of Sign Model, Eq(3), of Daily Changes in the VIX**

**Panel A: Coefficient Estimates**

Quantile	$\alpha$		$\beta$		$\lambda$		$\delta$	
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value
OLS	0.0505 <sup>a</sup>	0.0000	0.0186	0.1600	-0.0923 <sup>a</sup>	0.0000	-0.1022 <sup>a</sup>	0.0000
10%	0.0057 <sup>a</sup>	0.0000	0.0050	0.3300	-0.0924 <sup>a</sup>	0.0000	-0.1157 <sup>a</sup>	0.0000
20%	0.0113 <sup>a</sup>	0.0000	-0.0016	0.8400	-0.0774 <sup>a</sup>	0.0000	-0.1012 <sup>a</sup>	0.0000
30%	0.0186 <sup>a</sup>	0.0000	-0.0130	0.2000	-0.0706 <sup>a</sup>	0.0000	-0.0780 <sup>a</sup>	0.0000
40%	0.0258 <sup>a</sup>	0.0000	0.0027	0.8500	-0.0682 <sup>a</sup>	0.0000	-0.0829 <sup>a</sup>	0.0000
50%	0.0346 <sup>a</sup>	0.0000	0.0080	0.6800	-0.0685 <sup>a</sup>	0.0000	-0.0799 <sup>a</sup>	0.0000
60%	0.0458 <sup>a</sup>	0.0000	0.0091	0.6200	-0.0722 <sup>a</sup>	0.0000	-0.0730 <sup>a</sup>	0.0000
70%	0.0583 <sup>a</sup>	0.0000	-0.0032	0.8900	-0.0779 <sup>a</sup>	0.0000	-0.0423 <sup>c</sup>	0.0900
80%	0.0772 <sup>a</sup>	0.0000	0.0212	0.3600	-0.0906 <sup>a</sup>	0.0000	-0.0555 <sup>a</sup>	0.0300
90%	0.1120 <sup>a</sup>	0.0000	0.0051	0.8900	-0.1189 <sup>a</sup>	0.0000	-0.0163	0.6800

**Panel B: Autocorrelation Following a Positive Change ( $\beta$ ) versus Negative Change ( $\beta + \delta$ ) in the VIX**

Quantile	Positive Change	Negative Change
	$\beta$	$\beta + \delta$
	Coeff.	Coeff.
OLS	-	-0.1022
10%	-	-0.1157
20%	-	-0.1012
30%	-	-0.0780
40%	-	-0.0829
50%	-	-0.0799
60%	-	-0.0730
70%	-	-0.0423
80%	-	-0.0555
90%	-	-0.0163

Notes:

1. In Panel B only  $\beta$  and  $\delta$  that are significant, at a level of at least 10%, are used in the reported  $\beta + \delta$ ; insignificant  $\beta$  or  $\delta$  are treated as zero and “-” is used to indicate the case when both  $\beta$  and  $\delta$  are insignificant.
2. Significance levels are indicated by a for 1% level, b for 5% level, and c for 10% level.

**Table 5 Coefficient Estimates of Event Model, Eq(4), of Daily Changes in the VIX**

**Panel A: Post-Event One-Month Period**

Quantile	$\alpha$		$\beta$		$\lambda$		$\theta$		$\beta + \theta^1$
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.
OLS	0.002	0.012	0.019 <sup>a</sup>	0.000	0.016 <sup>b</sup>	0.027	-0.052	0.461	0.019
0.1	-0.067 <sup>a</sup>	0.000	-0.120 <sup>a</sup>	0.000	-0.003	0.781	-0.131 <sup>a</sup>	0.002	-0.251
0.2	-0.044 <sup>a</sup>	0.000	-0.105 <sup>a</sup>	0.000	-0.007	0.450	-0.128	0.544	-0.105
0.3	-0.028 <sup>a</sup>	0.000	-0.103 <sup>a</sup>	0.000	-0.009	0.317	-0.101	0.121	-0.103
0.4	-0.015 <sup>a</sup>	0.000	-0.084 <sup>a</sup>	0.000	-0.012	0.163	-0.130	0.171	-0.084
0.5	-0.003 <sup>a</sup>	0.000	-0.070 <sup>a</sup>	0.000	-0.002	0.876	-0.214	0.241	-0.070
0.6	0.009 <sup>a</sup>	0.000	-0.062 <sup>a</sup>	0.001	0.004	0.751	-0.184 <sup>b</sup>	0.027	-0.246
0.7	0.023 <sup>a</sup>	0.000	-0.055 <sup>a</sup>	0.001	0.025 <sup>b</sup>	0.035	-0.159	0.408	-0.055
0.8	0.043 <sup>a</sup>	0.000	-0.046 <sup>c</sup>	0.062	0.033	0.185	-0.140	0.756	-0.046
0.9	0.074 <sup>a</sup>	0.000	-0.024	0.301	0.056 <sup>a</sup>	0.001	0.317 <sup>a</sup>	0.000	0.317

**Panel B: Post-Event Three-Month Period**

Quantile	$\alpha$		$\beta$		$\lambda$		$\theta$		$\beta + \theta^1$
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.
OLS	0.002	0.016	0.019 <sup>a</sup>	0.000	0.007 <sup>c</sup>	0.084	-0.018	0.715	0.019
0.1	-0.067 <sup>a</sup>	0.000	-0.119 <sup>a</sup>	0.000	-0.007	0.300	-0.146 <sup>b</sup>	0.015	-0.265
0.2	-0.044 <sup>a</sup>	0.000	-0.108 <sup>a</sup>	0.000	-0.007	0.123	-0.042	0.450	-0.108
0.3	-0.028 <sup>a</sup>	0.000	-0.106 <sup>a</sup>	0.000	-0.007	0.098	-0.032	0.609	-0.106
0.4	-0.015 <sup>a</sup>	0.000	-0.089 <sup>a</sup>	0.000	-0.007	0.112	-0.007	0.882	-0.089
0.5	-0.003 <sup>a</sup>	0.000	-0.073 <sup>a</sup>	0.000	-0.004	0.394	0.012	0.742	-0.073
0.6	0.009 <sup>a</sup>	0.000	-0.064 <sup>a</sup>	0.000	0.002	0.794	-0.033	0.660	-0.064
0.7	0.023 <sup>a</sup>	0.000	-0.053 <sup>a</sup>	0.002	0.010	0.221	-0.074	0.232	-0.053
0.8	0.043 <sup>a</sup>	0.000	-0.047 <sup>c</sup>	0.066	0.019 <sup>b</sup>	0.017	-0.003	0.984	-0.047
0.9	0.074 <sup>a</sup>	0.000	-0.024	0.302	0.031 <sup>b</sup>	0.022	-0.072	0.543	-

Notes:

1. Only  $\beta$  and  $\theta$  that are significant, at a level of at least 10%, are used in the reported  $\beta + \gamma$ ; insignificant  $\beta$  or  $\gamma$  are treated as zero and “-” is used to indicate the case when both  $\beta$  and  $\gamma$  are insignificant.
2. Significance levels are indicated by a for 1% level, b for 5% level, and c for 10% level.

**Table 6 Coefficient Estimates of Event Model, Eq(5), of Daily Changes in the VIX for Economic Events**

**Part 1: 2007 Subprime Financial Crisis**

**Panel A: Post-Event One-Month Period**

Quantile	$\alpha$		$\beta$		$\lambda$		$\theta$		$\beta + \theta^1$
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.
OLS	0.002 <sup>a</sup>	0.008	0.019 <sup>a</sup>	0.000	0.021	0.044	-0.218 <sup>c</sup>	0.062	-0.199
0.1	-0.067 <sup>a</sup>	0.000	-0.120 <sup>a</sup>	0.000	0.008	0.264	-0.163 <sup>a</sup>	0.000	-0.283
0.2	-0.044 <sup>a</sup>	0.000	-0.108 <sup>a</sup>	0.000	-0.004	0.601	-0.088	0.200	-0.108
0.3	-0.028 <sup>a</sup>	0.000	-0.108 <sup>a</sup>	0.000	-0.004	0.618	-0.166	0.097	-0.108
0.4	-0.015 <sup>a</sup>	0.000	-0.085 <sup>a</sup>	0.000	-0.006	0.560	-0.044	0.641	-0.085
0.5	-0.003 <sup>a</sup>	0.000	-0.070 <sup>a</sup>	0.000	0.006	0.964	-0.252	0.861	-0.070
0.6	0.009 <sup>a</sup>	0.000	-0.062 <sup>a</sup>	0.001	0.029 <sup>b</sup>	0.046	-0.471 <sup>a</sup>	0.001	-0.533
0.7	0.024 <sup>a</sup>	0.000	-0.054 <sup>a</sup>	0.001	0.020 <sup>b</sup>	0.043	-0.320 <sup>a</sup>	0.000	-0.374
0.8	0.043 <sup>a</sup>	0.000	-0.046 <sup>c</sup>	0.064	0.043 <sup>b</sup>	0.015	-0.507 <sup>a</sup>	0.000	-0.553
0.9	0.075 <sup>a</sup>	0.000	-0.025	0.296	0.038 <sup>a</sup>	0.005	-0.642 <sup>a</sup>	0.000	-0.642

**Panel B: Post-Event Three-Month Period**

Quantile	$\alpha$		$\beta$		$\lambda$		$\theta$		$\beta + \theta^1$
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.
OLS	0.002 <sup>b</sup>	0.013 <sup>b</sup>	0.019 <sup>a</sup>	0.000	0.012 <sup>b</sup>	0.040	-0.073	0.248	0.019
0.1	-0.067 <sup>a</sup>	0.000	-0.116 <sup>a</sup>	0.000	-0.022 <sup>b</sup>	0.012	-0.080	0.116	-0.116
0.2	-0.044 <sup>a</sup>	0.000	-0.110 <sup>a</sup>	0.000	-0.009	0.317	-0.059	0.585	-0.110
0.3	-0.028 <sup>a</sup>	0.000	-0.109 <sup>a</sup>	0.000	-0.002	0.823	0.051	0.595	-0.109
0.4	-0.015 <sup>a</sup>	0.000	-0.090 <sup>a</sup>	0.000	0.001	0.917	0.019	0.611	-0.090
0.5	-0.003 <sup>a</sup>	0.000	-0.070 <sup>a</sup>	0.000	0.002	0.854	-0.009	0.835	-0.070
0.6	0.009 <sup>a</sup>	0.000	-0.061 <sup>a</sup>	0.001	0.026 <sup>a</sup>	0.007	-0.143 <sup>a</sup>	0.009	-0.204
0.7	0.023 <sup>a</sup>	0.000	-0.054 <sup>a</sup>	0.001	0.029 <sup>a</sup>	0.002	-0.188 <sup>a</sup>	0.002	-0.242
0.8	0.043 <sup>a</sup>	0.000	-0.041	0.102	0.036 <sup>a</sup>	0.004	-0.268 <sup>b</sup>	0.012	-0.309
0.9	0.074 <sup>a</sup>	0.000	-0.023	0.330	0.046 <sup>a</sup>	0.000	-0.184	0.222	-

Notes:

1. Only  $\beta$  and  $\theta$  that are significant, at a level of at least 10%, are used in the reported  $\beta + \gamma$ ; insignificant  $\beta$  or  $\gamma$  are treated as zero and “-” is used to indicate the case when both  $\beta$  and  $\gamma$  are insignificant.
2. Significance levels are indicated by a for 1% level, b for 5% level, and c for 10% level.

**Table 6 (continued)****Part 2: Eurozone Debt Crisis****Panel A: Post-Event One-Month Period**

Quantile	$\alpha$		$\beta$		$\lambda$		$\theta$		$\beta + \theta^1$
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.
OLS	0.002 <sup>a</sup>	0.005	0.019 <sup>a</sup>	0.000	-0.005	0.637	0.084	0.706	0.019
0.1	-0.067 <sup>a</sup>	0.000	-0.121 <sup>a</sup>	0.000	0.007	0.398	-0.154	0.503	-0.121
0.2	-0.044 <sup>a</sup>	0.000	-0.110 <sup>a</sup>	0.000	-0.006	0.352	-0.190	0.346	-0.110
0.3	-0.028 <sup>a</sup>	0.000	-0.109 <sup>a</sup>	0.000	-0.001	0.936	0.149	0.769	-0.109
0.4	-0.015 <sup>a</sup>	0.000	-0.089 <sup>a</sup>	0.000	0.000	0.996	0.215	0.762	-0.089
0.5	-0.004 <sup>a</sup>	0.000	-0.074 <sup>a</sup>	0.000	0.000	0.981	0.249 <sup>c</sup>	0.057	0.175
0.6	0.009 <sup>a</sup>	0.000	-0.068 <sup>a</sup>	0.000	-0.006	0.400	0.261 <sup>c</sup>	0.061	0.193
0.7	0.024 <sup>a</sup>	0.000	-0.060 <sup>a</sup>	0.000	-0.009	0.120	0.134	0.160	-0.060
0.8	0.043 <sup>a</sup>	0.000	-0.051 <sup>b</sup>	0.040	-0.022 <sup>a</sup>	0.000	0.136	0.226	-0.051
0.9	0.075 <sup>a</sup>	0.000	-0.027	0.257	-0.004	0.818	0.301	0.246	-

**Panel B: Post-Event Three-Month Period**

Quantile	$\alpha$		$\beta$		$\lambda$		$\theta$		$\beta + \theta^1$
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.
OLS	0.002 <sup>a</sup>	0.004	0.019 <sup>a</sup>	0.000	-0.005	0.418	-0.008	0.946	0.019
0.1	-0.068 <sup>a</sup>	0.000	-0.125 <sup>a</sup>	0.000	0.017 <sup>a</sup>	0.000	0.052	0.290	-0.125
0.2	-0.044 <sup>a</sup>	0.000	-0.113 <sup>a</sup>	0.000	0.001	0.791	0.102	0.242	-0.113
0.3	-0.028 <sup>a</sup>	0.000	-0.109 <sup>a</sup>	0.000	-0.004	0.424	0.120 <sup>b</sup>	0.014	0.011
0.4	-0.015 <sup>a</sup>	0.000	-0.089 <sup>a</sup>	0.000	-0.005	0.246	0.081	0.341	-0.089
0.5	-0.003 <sup>a</sup>	0.000	-0.074 <sup>a</sup>	0.000	-0.009 <sup>b</sup>	0.045	0.089	0.215	-0.074
0.6	0.010 <sup>a</sup>	0.000	-0.067 <sup>a</sup>	0.000	-0.011 <sup>b</sup>	0.018	0.011	0.875	-0.067
0.7	0.024 <sup>a</sup>	0.000	-0.058 <sup>a</sup>	0.001	-0.015 <sup>a</sup>	0.002	-0.027	0.684	-0.058
0.8	0.044 <sup>a</sup>	0.000	-0.050 <sup>b</sup>	0.044	-0.018 <sup>b</sup>	0.050	0.007	0.968	-0.050
0.9	0.075 <sup>a</sup>	0.000	-0.026	0.276	-0.020 <sup>a</sup>	0.003	-0.279 <sup>a</sup>	0.000	-0.279

Notes:

- Only  $\beta$  and  $\theta$  that are significant, at a level of at least 10%, are used in the reported  $\beta + \gamma$ ; insignificant  $\beta$  or  $\gamma$  are treated as zero and “-” is used to indicate the case when both  $\beta$  and  $\gamma$  are insignificant.
- Significance levels are indicated by a for 1% level, b for 5% level, and c for 10% level.

**Table 7 Coefficient Estimates of Event Model, Eq(4), of Daily Changes in the VIX for Geopolitical Events**

**Part 1: First Persian Gulf War**

**Panel A: Post-Event One-Month Period**

Quantile	$\alpha$		$\beta$		$\lambda$		$\theta$		$\beta + \theta^1$
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.
OLS	0.002 <sup>a</sup>	0.009	0.019 <sup>a</sup>	0.000	0.021 <sup>b</sup>	0.050	0.007	0.933	0.019
0.1	-0.067 <sup>a</sup>	0.000	-0.130 <sup>a</sup>	0.000	-0.054 <sup>a</sup>	0.000	0.211 <sup>a</sup>	0.000	0.081
0.2	-0.044 <sup>a</sup>	0.000	-0.113 <sup>a</sup>	0.000	-0.017	0.162	0.076 <sup>b</sup>	0.042	-0.037
0.3	-0.028 <sup>a</sup>	0.000	-0.109 <sup>a</sup>	0.000	-0.012	0.439	0.033	0.430	-0.109
0.4	-0.015 <sup>a</sup>	0.000	-0.086 <sup>a</sup>	0.000	0.003	0.801	-0.045	0.237	-0.086
0.5	-0.004 <sup>a</sup>	0.000	-0.072 <sup>a</sup>	0.000	0.001	0.891	-0.046	0.875	-0.072
0.6	0.009 <sup>a</sup>	0.000	-0.065 <sup>a</sup>	0.000	0.000	0.995	-0.108	0.011	-0.043
0.7	0.024 <sup>a</sup>	0.000	-0.056 <sup>a</sup>	0.001	0.001	0.946	-0.013	0.966	-0.056
0.8	0.043 <sup>a</sup>	0.000	-0.050 <sup>b</sup>	0.046	0.026	0.570	-0.085	0.929	-0.050
0.9	0.075 <sup>a</sup>	0.000	-0.027	0.257	0.097 <sup>a</sup>	0.000	0.217 <sup>a</sup>	0.001	0.217

**Panel B: Post-Event Three-Month Period**

Quantile	$\alpha$		$\beta$		$\lambda$		$\theta$		$\beta + \theta^1$
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.
OLS	0.002 <sup>a</sup>	0.009	0.019 <sup>a</sup>	0.000	0.007	0.261	-0.048	0.468	0.019
0.1	-0.067 <sup>a</sup>	0.000	-0.128 <sup>a</sup>	0.000	0.003	0.671	0.032	0.608	-0.128
0.2	-0.044 <sup>a</sup>	0.000	-0.108 <sup>a</sup>	0.000	0.004	0.396	-0.059	0.215	-0.108
0.3	-0.028 <sup>a</sup>	0.000	-0.105 <sup>a</sup>	0.000	-0.002	0.775	-0.041	0.834	-0.105
0.4	-0.015 <sup>a</sup>	0.000	-0.085 <sup>a</sup>	0.000	-0.001	0.866	-0.038	0.233	-0.085
0.5	-0.004 <sup>a</sup>	0.000	-0.072 <sup>a</sup>	0.000	-0.001	0.930	-0.031	0.864	-0.072
0.6	0.009 <sup>a</sup>	0.000	-0.062 <sup>a</sup>	0.001	0.000	0.953	-0.101 <sup>a</sup>	0.009	-0.163
0.7	0.024 <sup>a</sup>	0.000	-0.054 <sup>a</sup>	0.001	0.001	0.938	-0.061	0.793	-0.054
0.8	0.043 <sup>a</sup>	0.000	-0.047 <sup>c</sup>	0.058	0.003	0.763	-0.154	0.452	-0.047
0.9	0.075 <sup>a</sup>	0.000	-0.025	0.290	0.003	0.744	-0.202	0.520	-

Notes:

1. Only  $\beta$  and  $\theta$  that are significant, at a level of at least 10%, are used in the reported  $\beta + \gamma$ ; insignificant  $\beta$  or  $\gamma$  are treated as zero and “-” is used to indicate the case when both  $\beta$  and  $\gamma$  are insignificant.
2. Significance levels are indicated by a for 1% level, b for 5% level, and c for 10% level.

**Table 7 (continued)**

**Part 2: ISIS Declaration**

**Panel A: Post-Event One-Month Period**

Quantile	$\alpha$		$\beta$		$\lambda$		$\theta$		$\beta + \theta^1$
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coefficient
OLS	0.002 <sup>a</sup>	0.006	0.019 <sup>a</sup>	0.000	0.006	0.542	-0.346	0.010	-0.227
0.1	-0.067 <sup>a</sup>	0.000	-0.120 <sup>a</sup>	0.000	-0.005	0.706	-0.185 <sup>a</sup>	0.000	-0.305
0.2	-0.044 <sup>a</sup>	0.000	-0.109 <sup>a</sup>	0.000	0.009	0.229	-0.191	0.231	-0.109
0.3	-0.028 <sup>a</sup>	0.000	-0.102 <sup>a</sup>	0.000	-0.001	0.903	-0.147	0.252	-0.102
0.4	-0.015 <sup>a</sup>	0.000	-0.084 <sup>a</sup>	0.000	0.008	0.388	-0.325 <sup>a</sup>	0.000	-0.409
0.5	-0.003 <sup>a</sup>	0.000	-0.070 <sup>a</sup>	0.000	0.006	0.462	-0.360 <sup>a</sup>	0.006	-0.430
0.6	0.009 <sup>a</sup>	0.000	-0.061 <sup>a</sup>	0.001	0.004	0.740	-0.261 <sup>c</sup>	0.067	-0.322
0.7	0.024 <sup>a</sup>	0.000	-0.054 <sup>a</sup>	0.001	0.001	0.969	-0.424	0.504	-0.054
0.8	0.043 <sup>a</sup>	0.000	-0.049 <sup>b</sup>	0.048	0.007	0.716	-0.637 <sup>a</sup>	0.000	-0.676
0.9	0.075 <sup>a</sup>	0.000	-0.025	0.289	0.010	0.409	-0.256	0.216	-

**Panel B: Post-Event Three-Month Period**

Quantile	$\alpha$		$\beta$		$\lambda$		$\theta$		$\beta + \theta^1$
	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.	P-Value	Coeff.
OLS	0.002 <sup>a</sup>	0.006	0.019 <sup>a</sup>	0.000	0.001	0.924	-0.131	0.146	0.019
0.1	-0.067 <sup>a</sup>	0.000	-0.121 <sup>a</sup>	0.000	-0.005	0.655	-0.184 <sup>a</sup>	0.000	-0.305
0.2	-0.044 <sup>a</sup>	0.000	-0.109 <sup>a</sup>	0.000	0.006	0.302	-0.119	0.428	-0.109
0.3	-0.028 <sup>a</sup>	0.000	-0.102 <sup>a</sup>	0.000	-0.001	0.866	-0.148 <sup>c</sup>	0.067	-0.102
0.4	-0.015 <sup>a</sup>	0.000	-0.084 <sup>a</sup>	0.000	-0.004	0.423	-0.127	0.103	-0.084
0.5	-0.003 <sup>a</sup>	0.000	-0.070 <sup>a</sup>	0.000	-0.003	0.650	-0.132	0.368	-0.070
0.6	0.009 <sup>a</sup>	0.000	-0.065 <sup>a</sup>	0.000	-0.001	0.936	-0.019	0.957	-0.065
0.7	0.024 <sup>a</sup>	0.000	-0.055 <sup>a</sup>	0.001	-0.001	0.821	-0.010	0.889	-0.055
0.8	0.043 <sup>a</sup>	0.000	-0.049 <sup>b</sup>	0.047	-0.001	0.879	-0.069	0.193	-0.049
0.9	0.075 <sup>a</sup>	0.000	-0.027	0.277	0.005	0.783	-0.024	0.906	-

Notes:

1. Only  $\beta$  and  $\theta$  that are significant, at a level of at least 10%, are used in the reported  $\beta + \gamma$ ; insignificant  $\beta$  or  $\gamma$  are treated as zero and “-” is used to indicate the case when both  $\beta$  and  $\gamma$  are insignificant.
2. Significance levels are indicated by a for 1% level, b for 5% level, and c for 10% level.

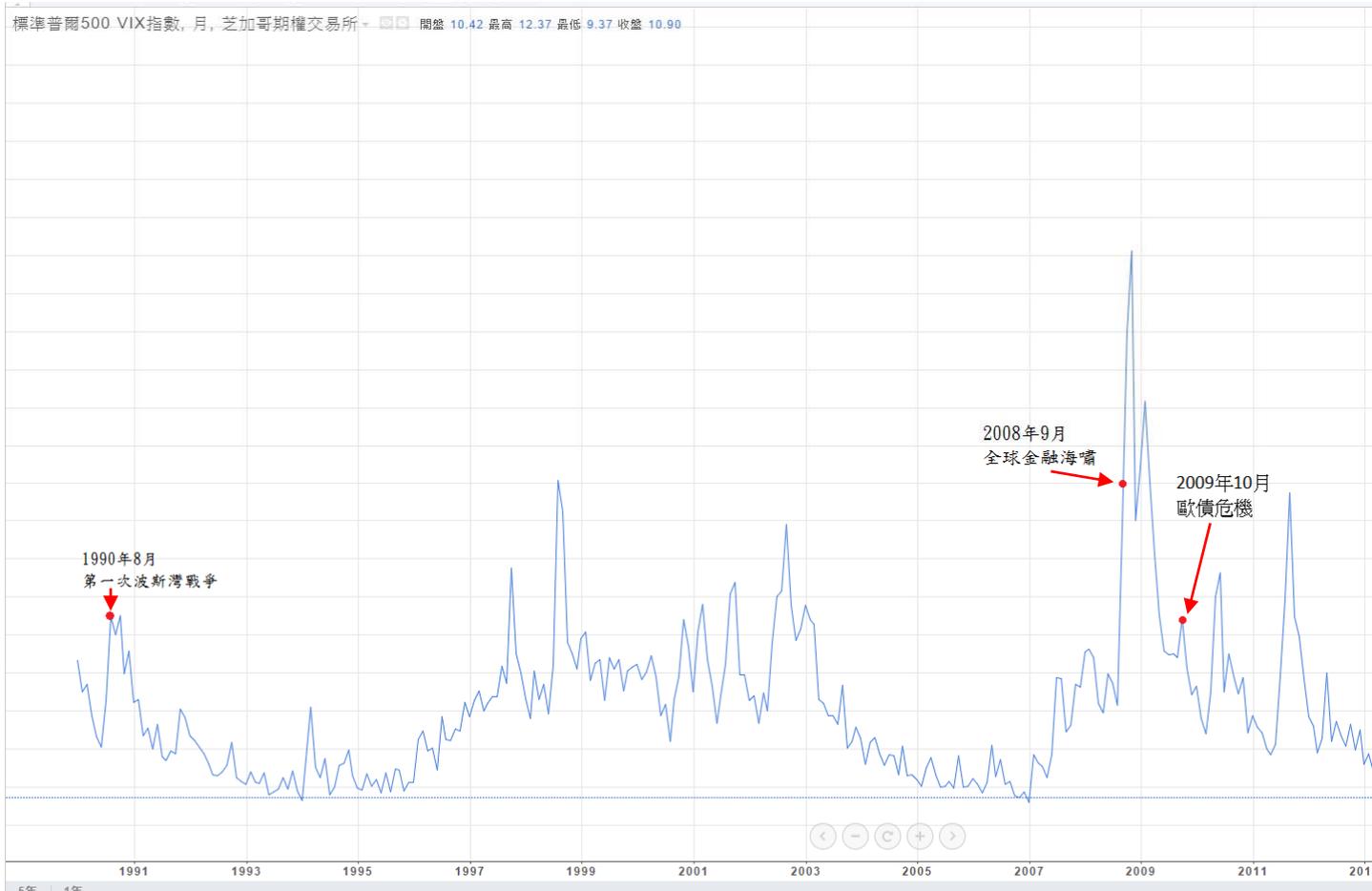
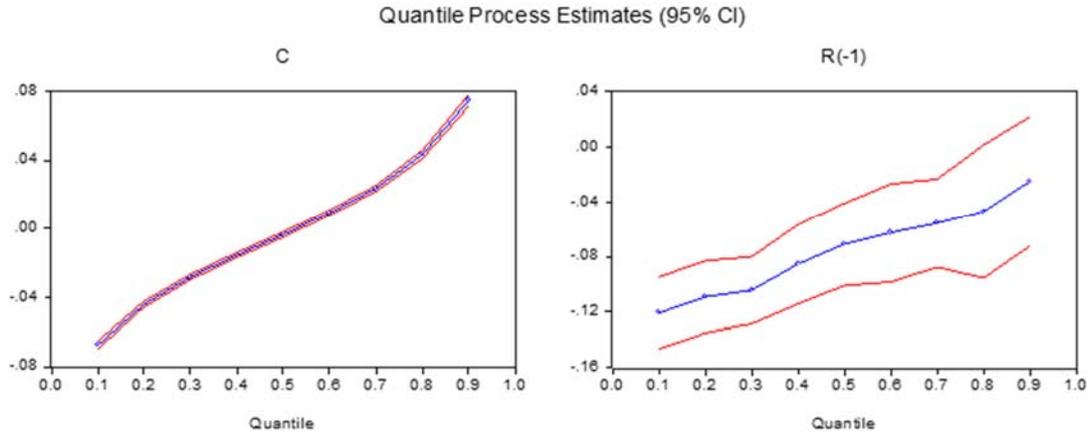
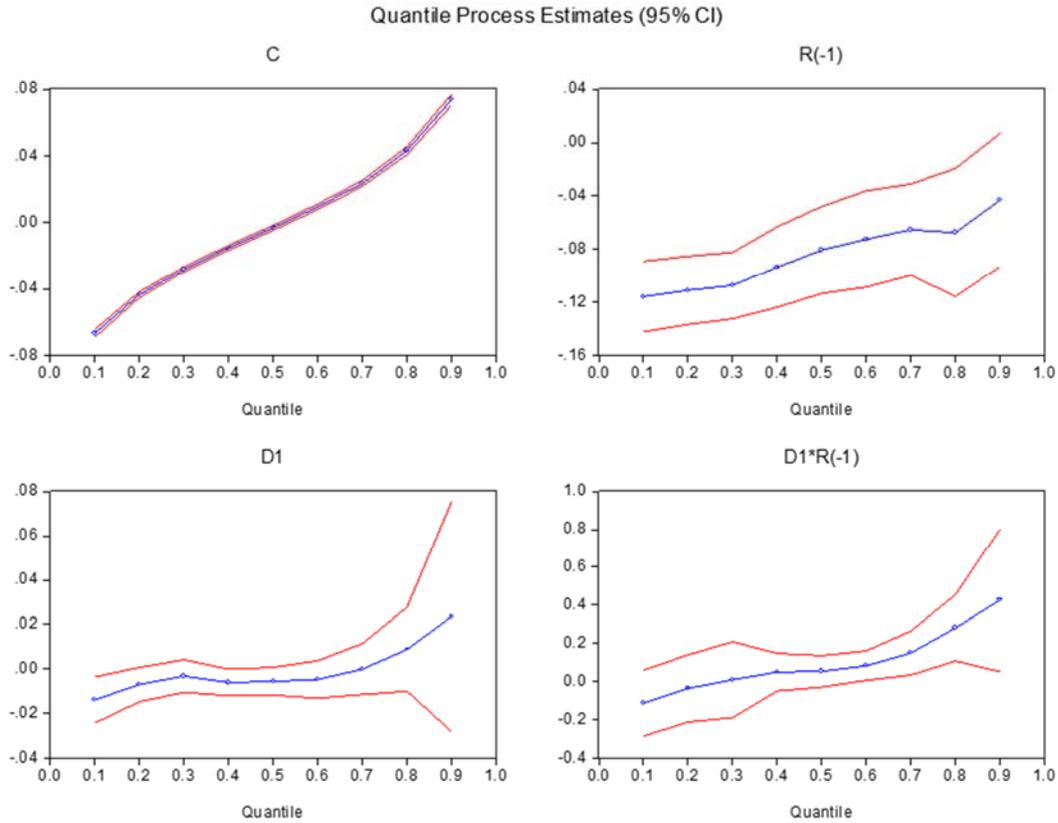


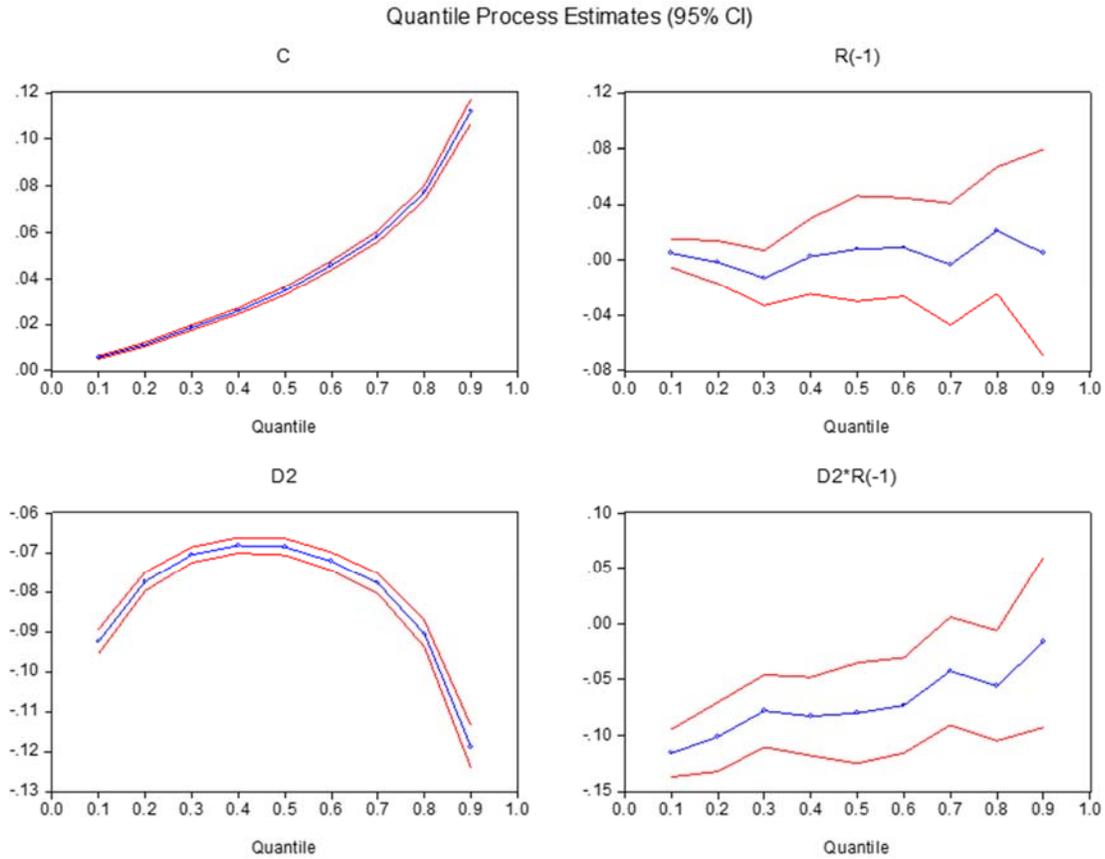
Fig 1 S&P500VIX, January 1990 to March 2017



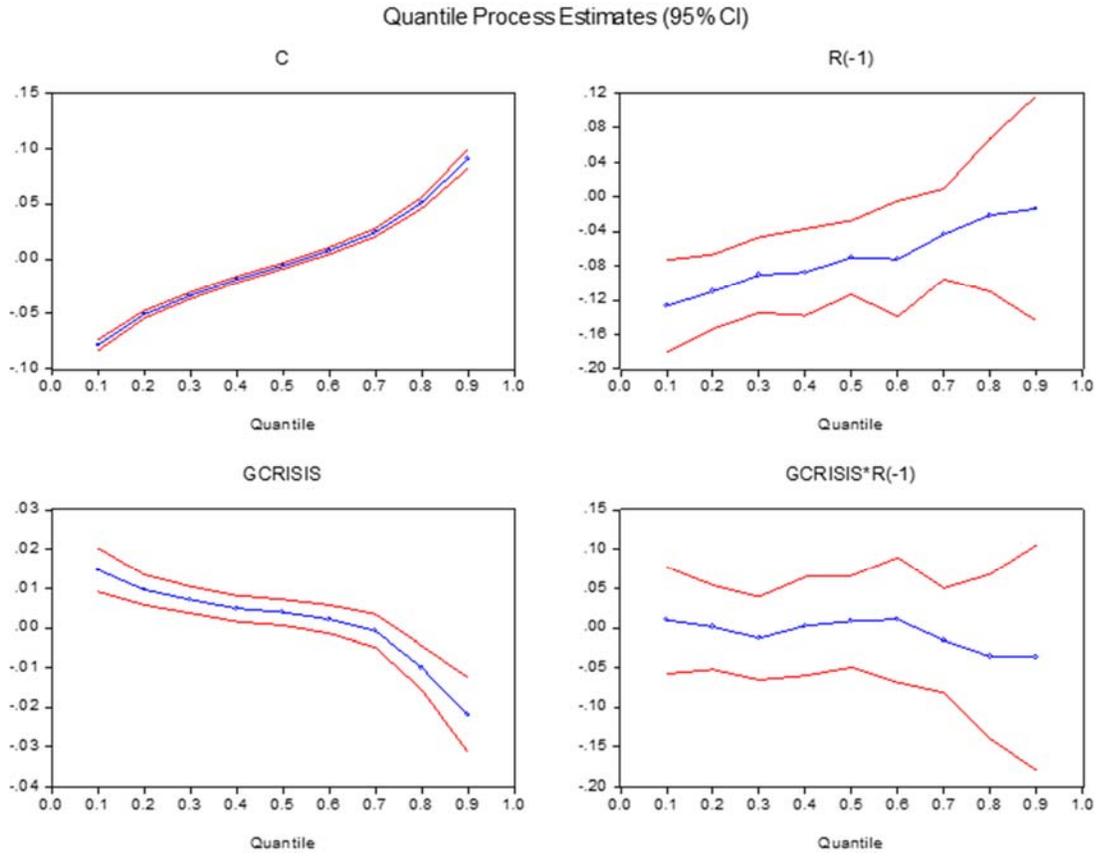
**Figure 1.** Estimated Coefficients,  $c$  and  $\beta_i(\tau)$ , of the basic QAR(1) model in Eq. (1) for the daily changes in the VIX. The dots on the blue line are the estimates for  $\tau$  quantile ranging from 0.1 to 0.9. The dots on the red lines represent the 95% confidence bands for the estimates.



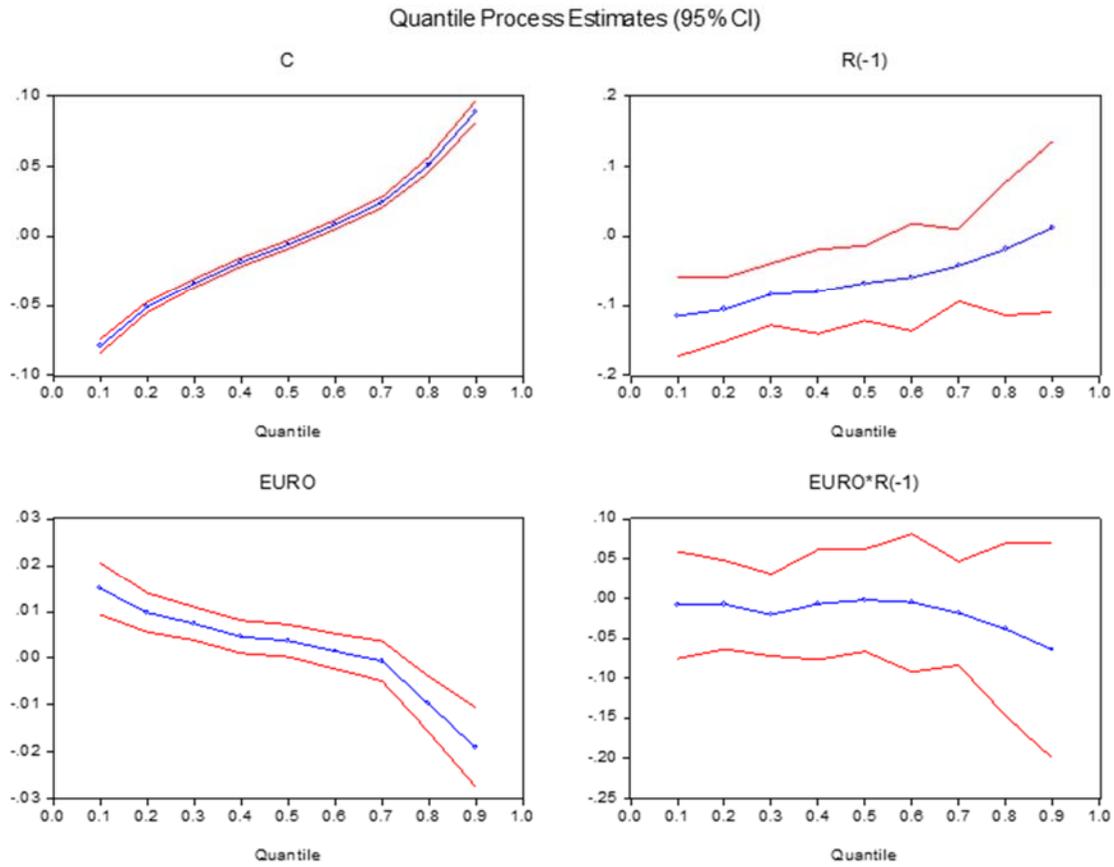
**Figure 2.** Estimated coefficients,  $c$ ,  $\beta_i(\tau)$ ,  $\lambda$ , and  $\gamma$ , of the Size QAR(1) model (Eq. (2)) for the daily changes in the VIX. The dots on the blue line are the estimates for  $\tau$  quantile ranging from 0.1 to 0.9. The dots on the red lines represent the 95% confidence bands for the estimates.



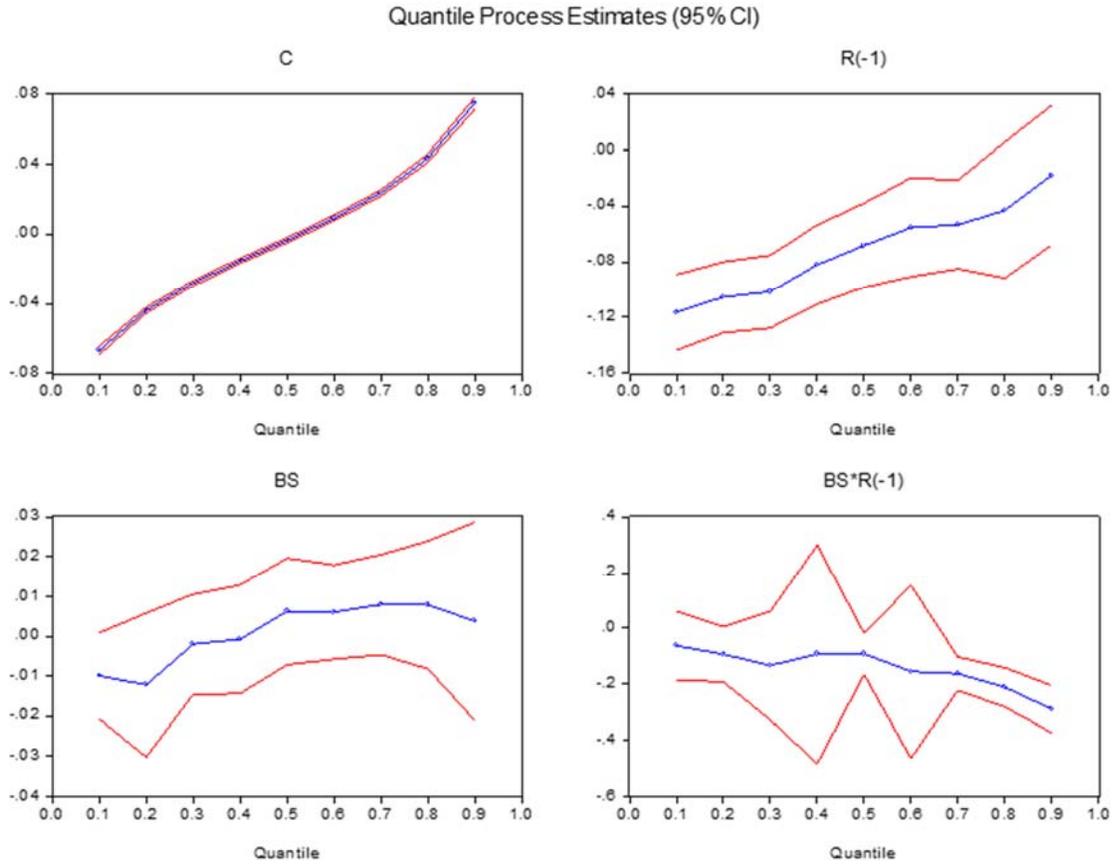
**Figure 3.** Estimated coefficients,  $c$ ,  $\beta_i(\tau)$ ,  $\lambda$ , and  $\delta$ , of the Sign QAR(1) model (Eq. (3)) for the daily changes in the VIX. The dots on the blue line are the estimates for  $\tau$  quantile ranging from 0.1 to 0.9. The dots on the red lines represent the 95% confidence bands for the estimates.



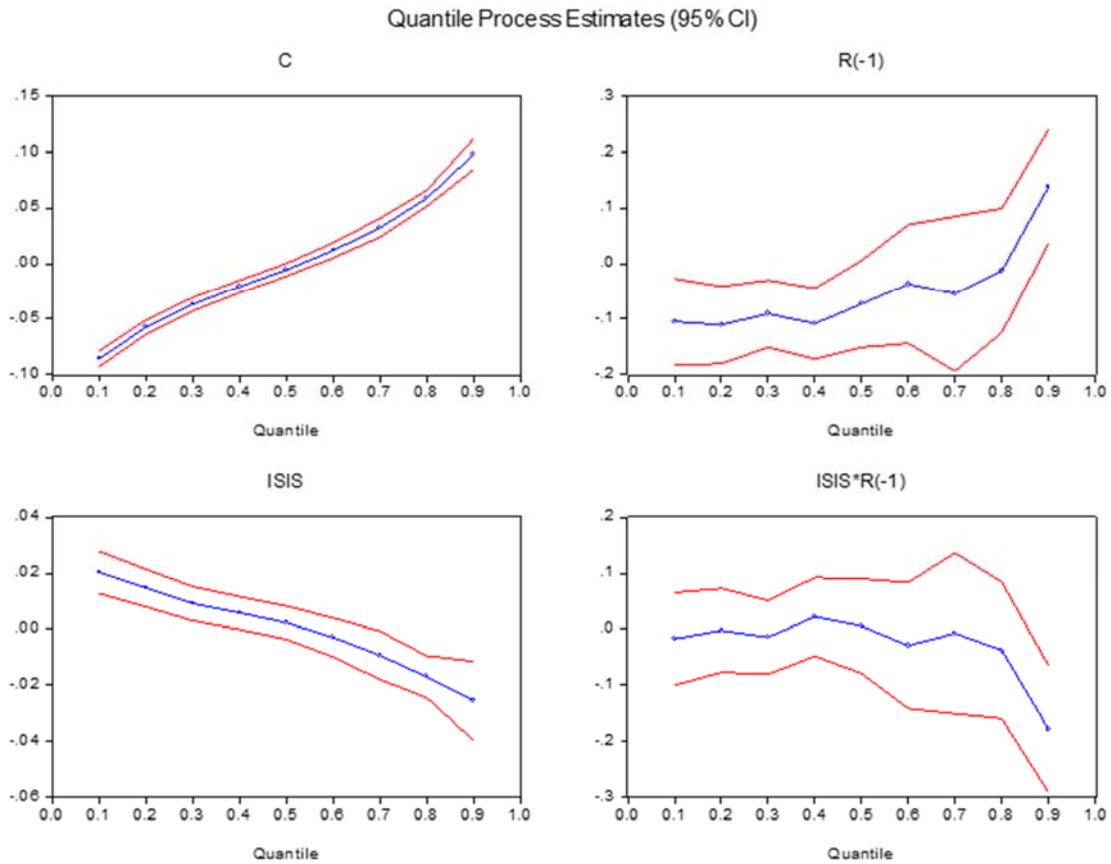
**Figure 4.** Estimated constant,  $c$ ,  $\beta_i(\tau)$ ,  $\lambda$ , and  $\theta$ , of the Event QAR(1) model, Eq. (4), for the daily changes in the VIX around the subprime crisis. The dots on the blue line are the estimates for  $\tau$  quantile ranging from 0.1 to 0.9. The dots on the red lines represent the 95% confidence bands for the estimates.



**Figure 5.** Estimated coefficients,  $c$ ,  $\beta_i(\tau)$ ,  $\lambda$ , and  $\theta$ , of the Event QAR(1) model, Eq. (4), for the changes in the VIX during the Euro Zone Debt Crisis. The dots on the blue line are the estimates for  $\tau$  quantile ranging from 0.1 to 0.9. The dots on the red lines represent the 95% confidence bands for the estimates.



**Figure 6** Estimated coefficients,  $c$ ,  $\beta_i(\tau)$ ,  $\lambda$ , and  $\theta$ , of the Event QAR(1) model, Eq. (4), for the daily changes in the VIX around the Persian Gulf War. The dots on the blue line are the estimates for  $\tau$  quantile ranging from 0.1 to 0.9. The dots on the red lines represent the 95% confidence bands for the estimates.



**Figure 7** Estimated coefficients,  $c$ ,  $\beta_1(\tau)$ ,  $\lambda$ , and  $\theta$ , of the Event QAR(1) model, Eq. (4), for the daily changes in the VIX around the ISIS Crisis. The dots on the blue line are the estimates for  $\tau$  quantile ranging from 0.1 to 0.9. The dots on the red lines represent the 95% confidence bands for the estimates.